**Economic order quantity**

**Economic order quantity** is the level of inventory that minimizes the total inventory holding costs and ordering costs. The framework used to determine this order quantity is also known as **Wilson EOQ Model**. The model was developed by F. W. Harris in 1913. But still R. H. Wilson is given credit for his early in-depth analysis of the model.

The Economic Order Quantity (EOQ) is the number of units that a company should add to inventory with each order to minimize the total costs of inventory—such as holding costs, order costs, and shortage costs. The EOQ is used as part of a continuous review inventory system, in which the level of inventory is monitored at all times, and a fixed quantity is ordered each time the inventory level reaches a specific reorder point. The EOQ provides a model for calculating the appropriate reorder point and the optimal reorder quantity to ensure the instantaneous replenishment of inventory with no shortages. It can be a valuable tool for small business owners who need to make decisions about how much inventory to keep on hand, how many items to order each time, and how often to reorder to incur the lowest possible costs.

The EOQ model assumes that demand is constant, and that inventory is depleted at a fixed rate until it reaches zero. At that point, a specific number of items arrive to return the inventory to its beginning level. Since the model assumes instantaneous replenishment, there are no inventory shortages or associated costs. Therefore, the cost of inventory under the EOQ model involves a trade-off between inventory holding costs (the cost of storage, as well as the cost of tying up capital in inventory rather than investing it or using it for other purposes) and order costs (any fees associated with placing orders, such as delivery charges). Ordering a large amount at one time will increase a small business's holding costs, while making more frequent orders of fewer items will reduce holding costs but increase order costs. The EOQ model finds the quantity that minimizes the sum of these costs.

The basic EOQ formula is TC = PD + HQ/2 + SD/Q

Where,

*TC* = the total inventory cost per year,

*PD* = the inventory purchase cost per year (price P multiplied by demand D in units per year),

H = the holding cost,

Q = the order quantity, and

S = the order cost (in dollars per order).

Breaking down the elements of the formula further, the yearly holding cost of inventory is H multiplied by the average number of units in inventory. Since the model assumes that inventory is depleted at a constant rate, the average number of units is equal to Q/2. The total orders cost per year is S multiplied by the number of orders per year, which is equal to the annual demand divided by the number of orders, or D/Q. Finally, PD is constant, regardless of the order quantity.

Taking these factors into consideration, solving for the optimal order quantity gives a formula of:

HQ/2 = SD/Q,

Or Q = √ (2DS/H).

The latter formula can be used to find the EOQ. For example, say that a painter uses 10 gallons of paint per day at $5 per gallon, and works 350 days per year. Under this scenario, the painter's annual paint consumption (or demand) is 3,500 gallons. Also assume that the painter incurs holding costs of $3 per gallon per year, and order costs of $15 per order. In this case, the painter's optimal order quantity can be found as follows: EOQ the square root of (2 3,500 15) /3 187 gallons. The number of orders is equal to D/Q, or 3,500 / 187. Thus the painter should order 187 gallons about 19 times per year, or every three weeks or so, in order to minimize his inventory costs.

The EOQ will sometimes change as a result of quantity discounts, which are provided by some suppliers as an [incentive](http://www.answers.com/topic/incentive" \t "_top) for customers to place larger orders. For example, a certain supplier may charge $20 per unit on orders of less than 100 units and only $18 per unit on orders over 100 units. To determine whether it makes sense to take advantage of a quantity discount when reordering inventory, a small business owner must compute the EOQ using the formula (Q =√( 2DS/H)), compute the total cost of inventory for the EOQ and for all price break points above it, and then select the order quantity that provides the minimum total cost.

For example, say that the painter can order 200 gallons or more for $4.75 per gallon, with all other factors in the computation remaining the same. He must compare the total costs of taking this approach to the total costs under the EOQ. Using the total cost formula outlined above, the painter would find TC PD HQ/2 SD/Q (5 3,500) (3 187)/2 + (15 3,500)/187 $18,062 for the EOQ. Ordering the higher quantity and receiving the price discount would yield a total cost of (4.75 3,500) (3 200)/2 (15 3,500)/200 $17,187. In other words, the painter can save $875 per year by taking advantage of the price break and making 17.5 orders per year of 200 units each.

Mathematical model that determines the amount of goods to order to meet projected demand while minimizing inventory costs. In the original version of the model, demand is assumed to be known and constant throughout the year. Ordering cost is assumed to be a fixed amount per order, and carrying costs are assumed to be constant per unit. EOQ is computed as

http://content.answers.com/main/content/img/barrons/accounting/tables/EconomicOrderQuantity.jpg

If the carrying cost is expressed as a percentage of average inventory value (say, 12% per year to hold inventory), then the denominator value in the EOQ formula would be 12% times the price of an item.

**Underlying assumptions**

1. The ordering cost is constant.
2. The rate of demand is constant
3. The lead time is fixed
4. The purchase price of the item is constant i.e no discount is available
5. The replenishment is made instantaneously, the whole batch is delivered at once.

EOQ is the quantity to order, so that ordering cost + carrying cost finds its minimum. (Common misunderstanding is, that formula tries to find when these are equal.)

**Variables**

* *Q* = order quantity
* *Q* \* = optimal order quantity
* *D* = annual demand quantity of the product
* *P* = purchase cost per unit
* *C* = fixed cost per order (*not* per unit, in addition to unit cost)
* *H* = annual holding cost per unit (also known as carrying cost) (warehouse space, refrigeration, insurance, etc. usually not related to the unit cost)

**The Total Cost function**

The single-item EOQ formula finds the minimum point of the following cost function: EOQ is the level of the inventory where ordering cost and carrying cost remains equal.

***Total Cost = purchase cost + ordering cost + holding cost***

- Purchase cost= the variable cost of goods= purchase unit price × annual demand quantity=P×D.

- Ordering cost=the cost of placing orders= C × D/Q (each order has a fixed cost C, and we need to order D/Q times per year.)

- Holding cost= H × Q/2(the average quantity in stock (between fully replenished and empty) is Q/2)

TC = PD + {\frac{CD}{Q}} + {\frac{HQ}{2}}

To determine the minimum point of the total cost curve, set its derivative equal to zero:

{\frac{dTC(Q)}{dQ}} = {\frac{d}{dQ}}\left(PD + {\frac{CD}{Q}} + {\frac{HQ}{2}}\right)=0.

The result of this derivation is:

-{\frac{CD}{Q^2}} + {\frac{H}{2}}=0.

Solving for Q gives Q\* (the optimal order quantity):

{\frac{H}{2}}={\frac{CD}{Q^2}}

Q^2={\frac{2CD}{H}}

Therefore, Q^* = \sqrt{\frac{2CD}{H}} .

Note that interestingly, Q\* is independent of P, it is a function of only C, D, H.